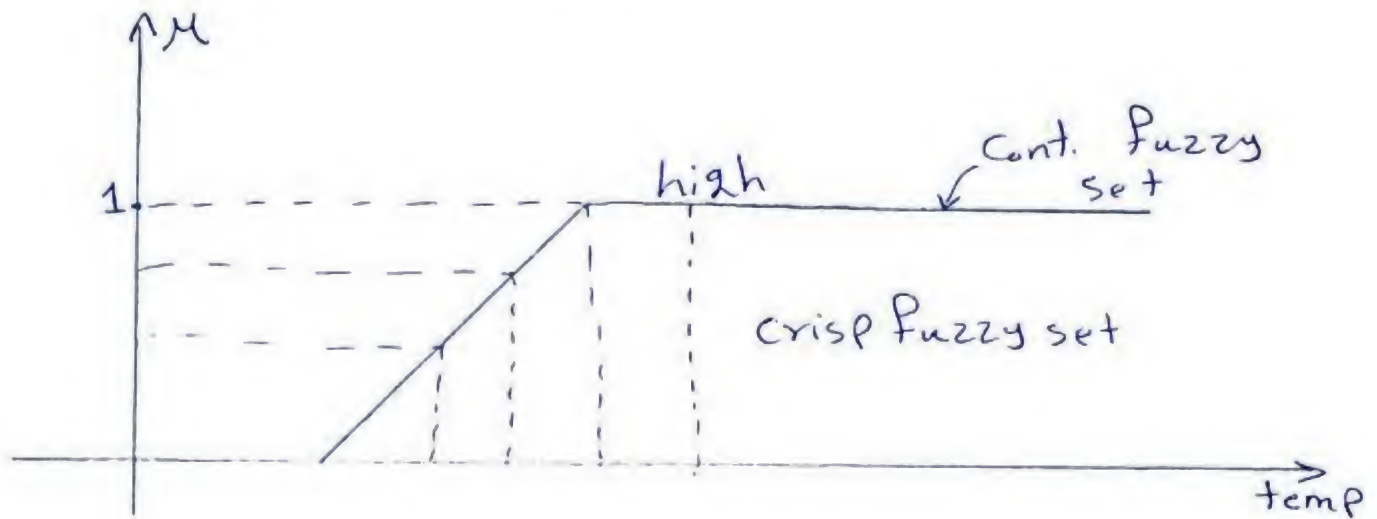


Fuzzy sets:

- Crisp Fuzzy set (discrete)
- Cont. Fuzzy sets



\* The crisp Fuzzy sets:

→ For any crisp Fuzzy sets: A "label for Fuzzy set"  
we can write the mathematical form to represent  
"A" as:

$$A = \{ (x_i, \mu_A(x_i)) \mid x_i \in X \}$$

$i = 1, 2, \dots, n$

where:

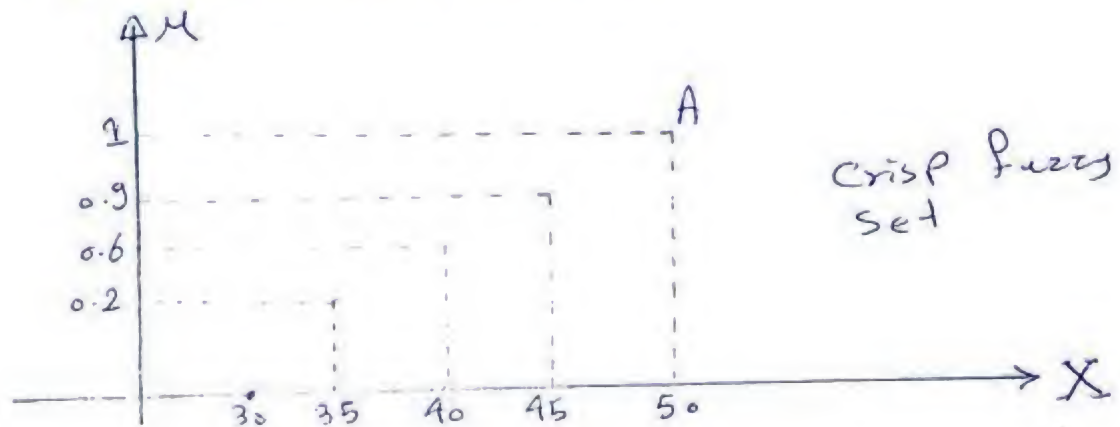
- $x_i$  → the elements of the crisp Fuzzy sets  
or the member of the " " " "
- $n$  → no. of the elements in the crisp Fuzzy set.

$X$ : is the universe of discourse.  
 [جميع القيم المتاحة لا (Fuzzy set) التي من الممكن الاختيار منها]

\* Universe of discourse

↳ range of all possible values considered to Fuzzy sets. (القيم المتاحة على المحور الأفقي)

-  $\mu_A(x_i) \rightarrow$  degree or grade of the belonging  $x_i$  to the Fuzzy set  $A$  ( $0 \rightarrow 1$ )



\* Method (1)

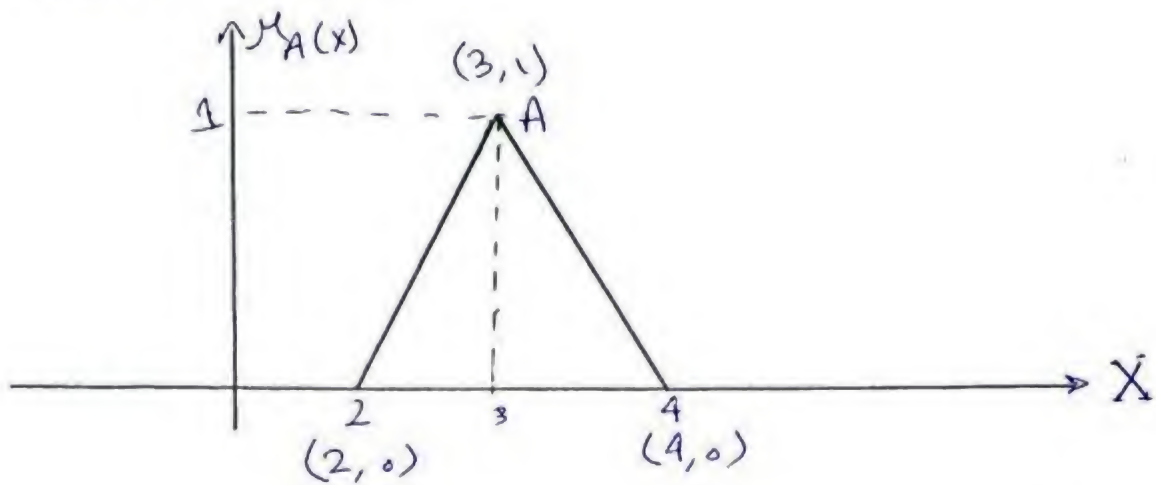
$$A = \{ \underbrace{(30, 0)}_{\text{element}}, \underbrace{(35, 0.2)}_{\mu}, (40, 0.6), (45, 0.9), (50, 1) \}$$

\* Method (2)

$$\left\{ \sum_{i=1}^n \mu_A(x_i) / x_i \mid x_i \in X \right\}$$

$$A = \left\{ \frac{0}{30}, \frac{0.2}{35}, \frac{0.6}{40}, \frac{0.9}{45}, \frac{1}{50} \right\}$$

note: method 1 & 2 are being used to describe Fuzzy sets.



⇒ to describe cont. Fuzzy sets

1) Graphical (previous figure)

$$2) \mu_A(x) = \begin{cases} \dots & 2 \leq x \leq 3 \\ \dots & 3 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

$$\hookrightarrow \mu_A = \begin{cases} x-2 & 2 \leq x \leq 3 \\ -x+4 & 3 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

$$\rightarrow \frac{\mu_A - 0}{x-2} = \frac{1-0}{3-2} \Rightarrow \mu_A = x-2$$

$$\rightarrow \frac{\mu_A - 0}{x-4} = \frac{1-0}{3-4} \Rightarrow \mu_A = -x+4$$

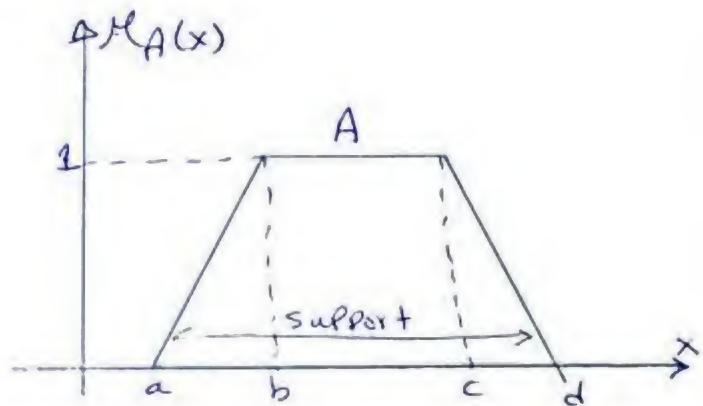


\* The most common type is the cont. Fuzzy sets:-

⇒ other concepts about Fuzzy sets:-

1) Support:-

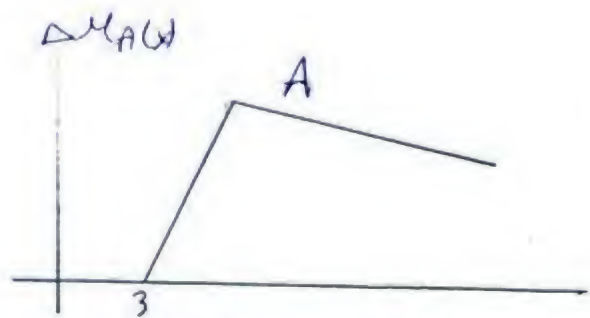
$$\text{support}(A) = [a, d]$$



↳ the elements (members) of Fuzzy set where its MF (membership Function) degree

$$\neq 0 \quad [\mu_A(x) \neq 0]$$

$$\text{support}(A) = [3, \infty]$$

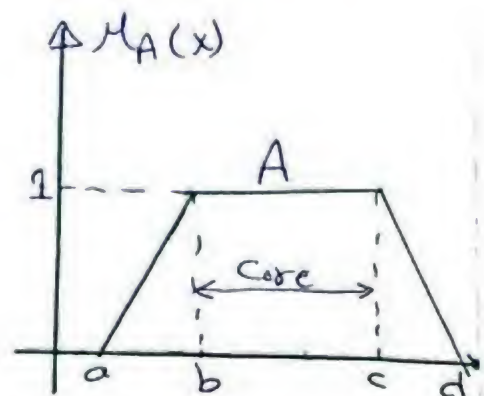


2] Core:-

$$\text{core}(A) = [b, c]$$

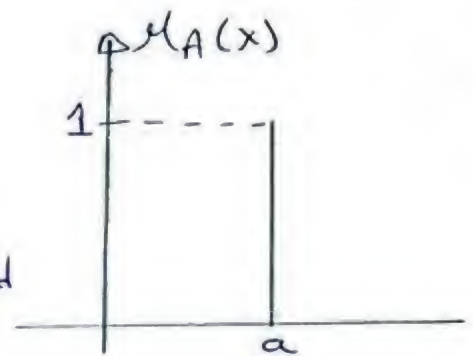
→ the elements of Fuzzy set where its MF degree = 1

$$(\mu_A(x) = 1)$$



### 3] Singleton

→ when the no. of elements of fuzzy set is equal to 1 with  $\mu = 1$ , the fuzzy set is called singleton fuzzy set.

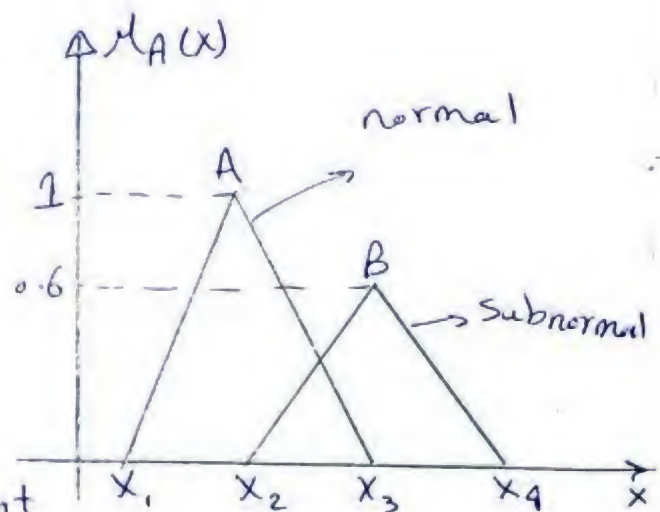


### "classification of fuzzy sets"

#### 1] Normal Fuzzy sets & subnormal fuzzy sets ::

Normal Fuzzy set :

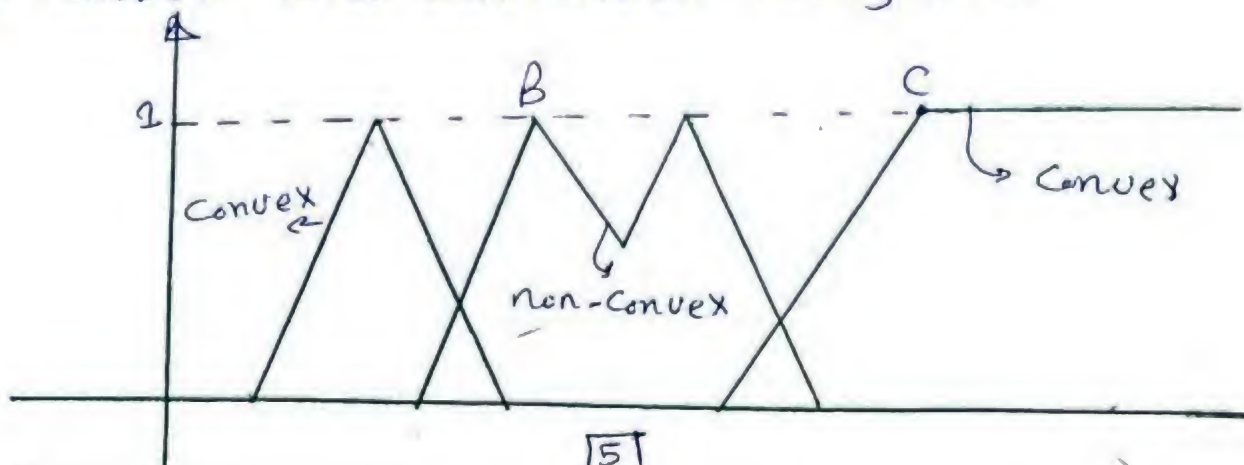
contains at least one element with  $\mu_A = 1$



Subnormal Fuzzy set

doesn't contain any element with  $\mu = 1$

#### 2) Convex and non Convex Fuzzy sets

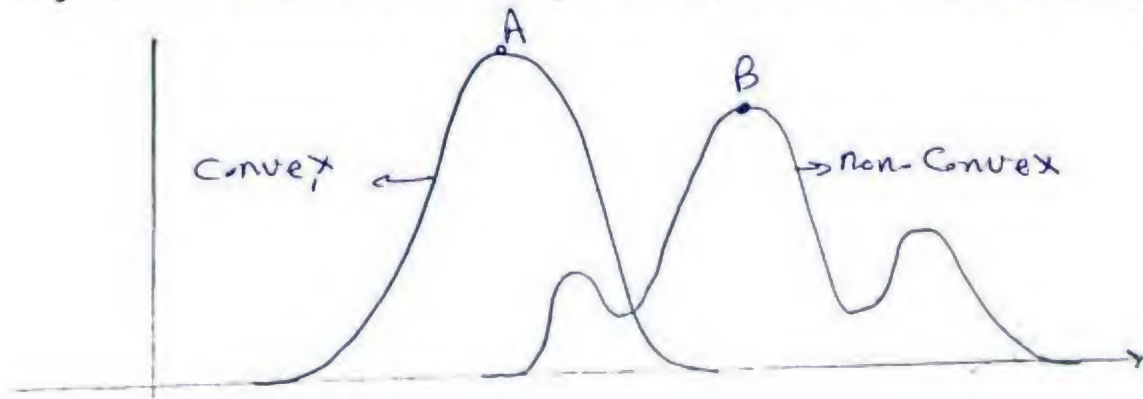




← في الشكل السابق :

A ←  $\mu$  زادت من 0 ← 1 ولما قلت، بتقل بشكل مرتب من 1 ← 0.

B ← زادت من 0 ← 1 ولكن قلت وموحيش للغير.



### \* Convex Fuzzy sets

↳ Fuzzy set is convex when its  $\mu$  is monotonically increasing or decreasing or increasing and decreasing over the elements of the set.

→ range must be from 0 → 1 to be normal fuzzy set.

\* In design process of Fuzzy Controller, we need Fuzzy sets to be:

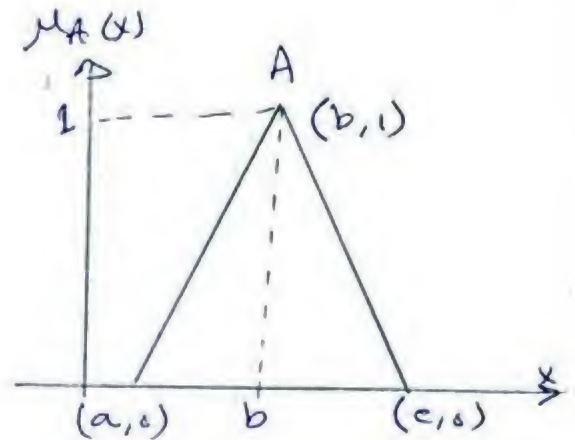
- 1) Normal Fuzzy sets.
- 2) Convex " " .
- 3) has bounded Support.

## "Common Fuzzy sets"

### [1] Triangular Fuzzy set:-

← من أشهر النواع، سهل التعريف.

↳ has 3 tuning Parameters that control the shape of the Fuzzy set.



$$a \leq b \leq c$$

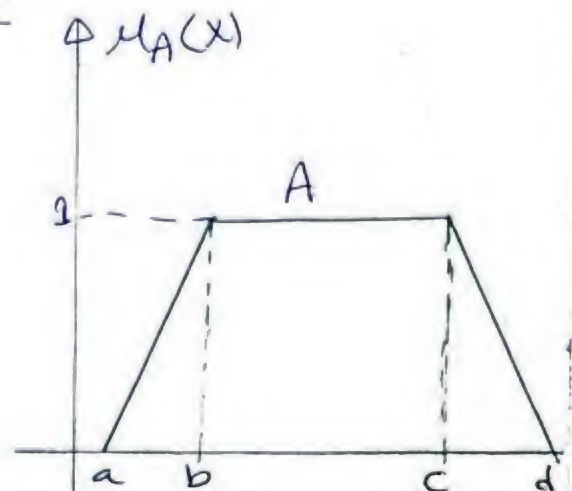
$$\mu_A(x) = \begin{cases} \frac{x-a}{b-a} & a \leq x \leq b \\ \frac{x-c}{b-c} & b \leq x \leq c \\ 0 & \text{otherwise} \end{cases}$$

$$\mu_A(x) = \max \left[ \min \left( \frac{x-a}{b-a}, \frac{x-c}{b-c} \right), 0 \right]$$

### [2] Trapezoidal Fuzzy set

↳ has 4 tuning Parameters that control the Fuzzy Set.

$$a \leq b \leq c \leq d$$



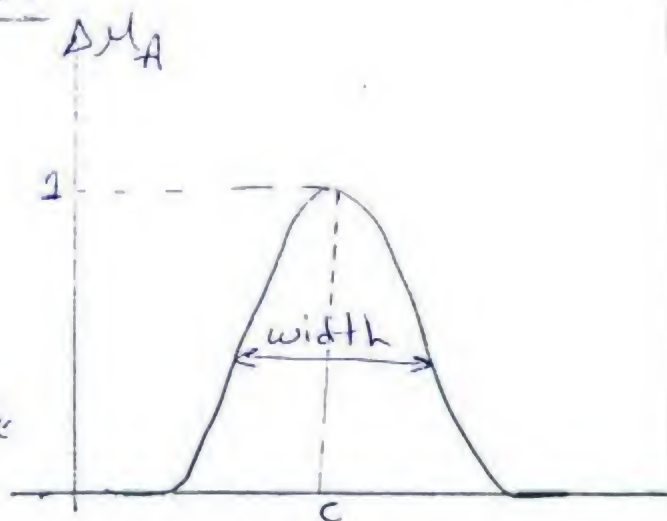
$$\mu_A(x) = \begin{cases} \frac{x-a}{b-a} & a \leq x \leq b \\ 1 & b \leq x \leq c \\ \frac{x-d}{c-d} & c \leq x \leq d \\ 0 & \text{otherwise} \end{cases}$$

$$\mu_A(x) = \max\left[\min\left(\frac{x-a}{b-a}, 1, \frac{x-d}{c-d}\right), 0\right]$$

### [3] Gaussian Fuzzy set

↳ has 2 tuning Parameters  
that control Fuzzy set  
(c, w)

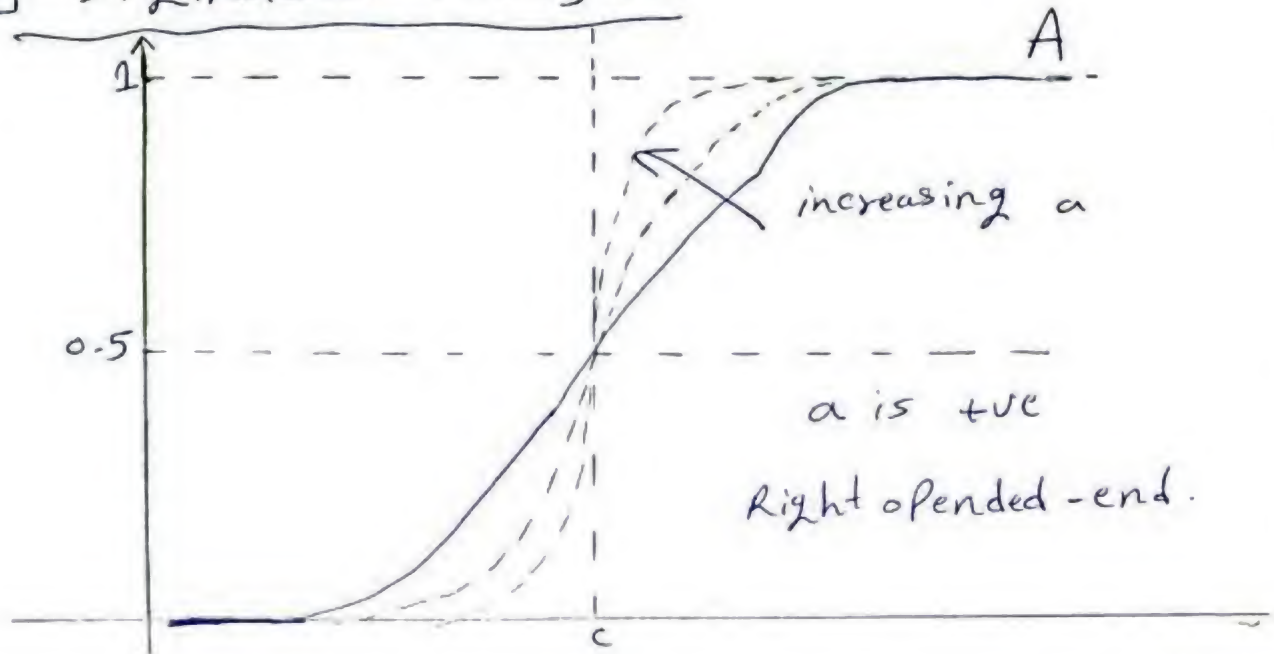
~~w is width of~~  
w → control width of shape



$$\mu_A(x) = e^{-0.5 \left(\frac{x-c}{w}\right)^2}$$



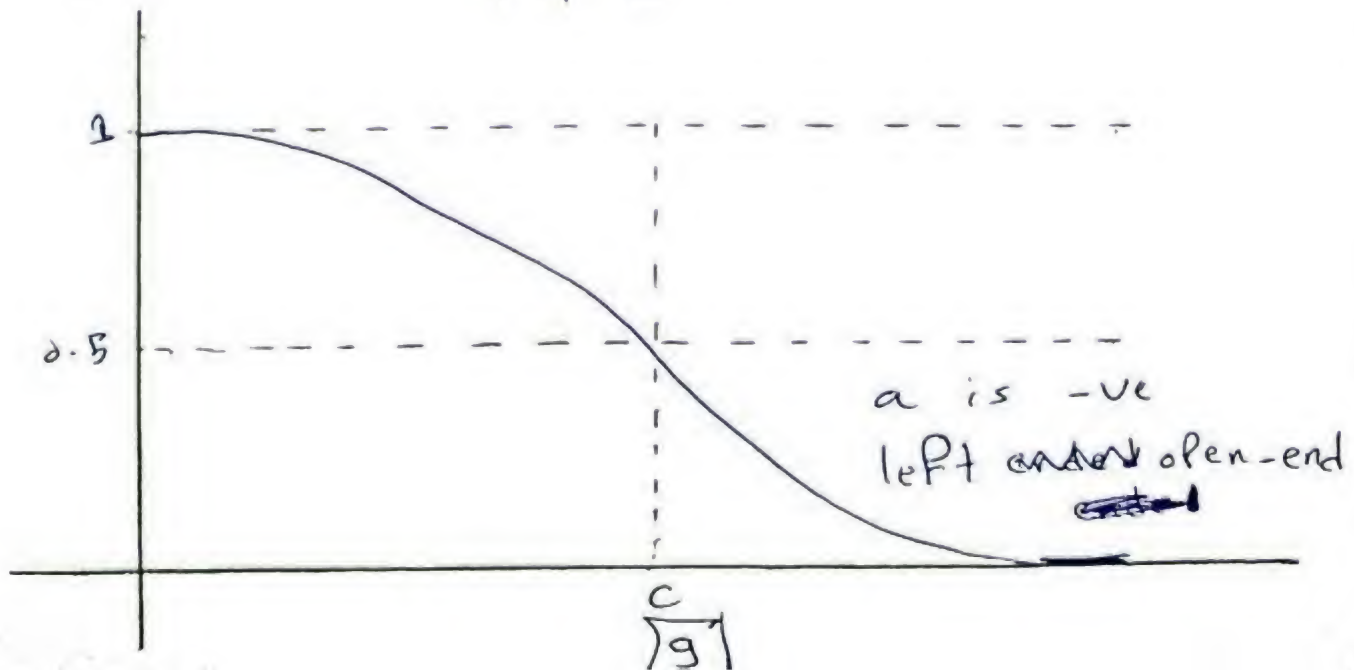
#### [4] Sigmoidal Fuzzy set



→ has 2 tuning Parameters that control the shape of the fuzzy sets  $a, c$ .

→  $a$  control the slope of this shape.

$$\mu_A(x) = \frac{1}{1 + e^{-a(x-c)}}$$



→ the sign of  $a$  determine the open-end

of the shape  $\begin{cases} \rightarrow +ve \text{ (Right open end)} \\ \rightarrow -ve \text{ (left open end)} \end{cases}$

← شكل الأوامر على المثال :

- $\text{gauss}^{mf}(x, [w, c])$
- $\text{trap}^{mf}(x, [a \ b \ c \ d])$
- $\text{trimf}(x, [a \ b \ c])$
- $\text{sigmf}(x, [a \ c])$

### "operations of Fuzzy sets"

1) union operation (Connective OR operator)

→ union of two fuzzy sets  $A, B$  is denoted

by :  $C = A \cup B$

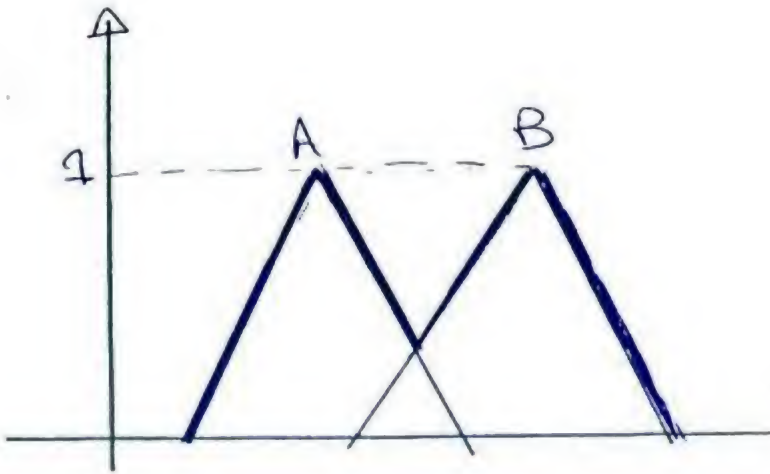
where:

$\mu_C(x) \begin{cases} \rightarrow \text{Max-operation} \\ \mu_C(x) = \max(\mu_A(x), \mu_B(x)) \end{cases}$

$\rightarrow \text{Product operation}$

$\mu_C(x) = \mu_A(x) + \mu_B(x) - \mu_A(x) \cdot \mu_B(x)$

Ex



في الخط والتعبير  
د. د. د. د.  
 $A \cup B$

EX → The fuzzy set A = "high temp." defined as:

$$A = \left\{ \frac{0}{36.5} + \frac{0}{37} + \frac{0.1}{37.5} + \frac{0.5}{38} + \frac{0.8}{38.5} + \frac{1}{39} + \frac{1}{39.5} + \frac{1}{40} \right\}$$

and the Fuzzy set  $B = \text{"Dangerous temp"}$  defined as

$$B = \left\{ \frac{0}{37.5}, \frac{0.1}{38} + \cancel{0.2} \frac{0.2}{38.5} + \frac{0.5}{39} + \frac{1}{40} + \frac{0.8}{39.5} \right\}$$

Find  $A \cup B$  = high or dang. temp

→ using max operation

$$C = A \cup B = \left\{ \frac{0}{36.5} + \frac{0}{37} + \frac{0.1}{37.5} + \frac{0.5}{38} + \frac{0.8}{38.5} + \frac{1}{39} + \frac{1}{39.5} + \frac{1}{40} \right\}$$

$$C = A \cup B$$

using product operation

$$= \left\{ \frac{0}{36.5} + \frac{0}{37} + \frac{0.1}{37.5} + \frac{0.1}{37.5} + \frac{0.55}{38} + \dots \right\}$$

له صمدی یدخل ارقام مختلفه



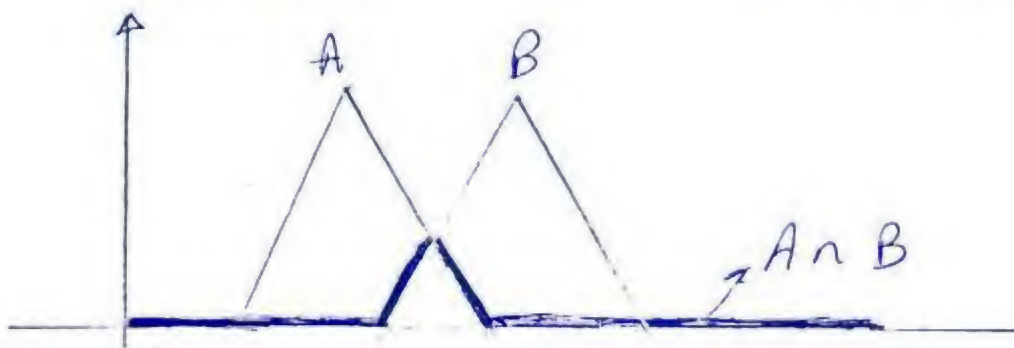
## [2] Intersection operation (And Connective operator)

↳ the intersection of two fuzzy sets  $A$  and  $B$  is defined as:

$$C = A \cap B$$

Where:

- minimum rule  $\Rightarrow \min(\mu_A(x), \mu_B(x))$
- product rule  $\Rightarrow \mu_C(x) = \mu_A(x) \cdot \mu_B(x)$



## [3] The Complement operation (Not operation)

↳ Complement of Fuzzy set  $A$  is denoted as:  
 $\bar{A}$  or  $\sim A$  represent to what degree the element doesn't belong to Fuzzy set.

$$\mu_{\bar{A}}(x) = 1 - \mu_A(x)$$

